Some General Properties of the Inverse Shadowing Property

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Submission date:- 7/3/2018 Acceptance date:- 8/4/2018 Publication date:- 2/12/2018

Keywords: Pseudo-orbit, Shadowing property, Inverse Shadowing property.

Abstract

The inverse shadowing property are concentrated , it has important properties and applications in maths . In this paper some general properties of this concept are proved . Let $(\mathcal{X}, d)$ be a metric space $f, g : (\mathcal{X}, d) \to (\mathcal{X}, d)$ be maps have the inverse shadowing property. We show the maps $g \circ f$, f^k, $f \approx_h g$, $f \times g$, have the inverse shadowing property.

If f and $g : (\mathbb{R}^n, d) \to (\mathbb{R}^n, d)$ are maps on a metric space $(\mathbb{R}^n, d)$ have the inverse shadowing property, We show the maps $f + g$ and $f . g$ have the inverse shadowing property.

1- Introduction

The pseudo-orbits (approximated orbits) are Nowadays , the question about the existing of a correct orbit almost a pseudo-orbit is elevated and powerfully considered. It appears that the better method to carry out this thoughts it before the concept of the property of (directly) shadowing. The concept of shadowing, in fact, plays a significant place in realization the asymptotic behavior of dynamical systems; this goes back to 1960s, the work in [1],[2] the inverse idea is also important, it He , each correct orbit of the system can be approximated by a pseudo orbit for particular properties. Virtually , those pseudo orbits are passable for a retroactive assigned category of orbits created by continuing maps. This one concept is invite inverse shadowing, which was introduce in [3] ,[4] using δ method. In [5] thoughtful the hyperbolic homeomorphisms on compact manifolds and presented inverse shadowing with regard to a category of δ methods which are represented by continuous maps from the manifold into the space of inverse infinite sequences in the manifold with the product topology. In [6] studies the bi-shadowing properties , he proved some general properties of the bi-shadowing properties .In [7] study the inverse shadowing property for actions of some finitely generated groups . A tube condition for such actions is introduced and analyses . we prove a reductive inverse shadowing theorem for actions of virtually nilpotent groups .

In [8] showed some result on the chaotic properties of the shadowing property.

In this paper, some preliminaries needed are given, also we state and prove some general properties and theorems about the inverse shadowing property.

2- Preliminaries:

Let $f : (\mathcal{X}, d) \longrightarrow (\mathcal{X}, d)$ be a map defined on a metric space $(\mathcal{X}, d)$ and consider the dynamical system on $\mathcal{X}$ generated by the iteration of f, that is $f^0 = \mathrm{id}_{\mathcal{X}}$ and $f^{n+1} = f^n \circ f$, for all $n \in \mathbb{Z}$. We shall identify the map f with the corresponding dynamical system. A sequence $\{x_n\}_{n=0}^{\infty} \subset \mathcal{X}$ is said to be a (true) orbit of f if $x_{n+1} = f(x_n)$, for all $n \in \mathbb{Z}$. A sequence $\{x_n\}_{n=0}^{\infty} \subset \mathcal{X}$ is called a δ-pseudo-orbit of f if $d(f(x_n), x_{n+1}) \leq \delta$, for all $n \in \mathbb{Z}$ and for $\delta > 0$.

We say that a point $x \in \mathcal{X}$ ϵ −shadows a δ −pseudo orbit $\{x_i\}_{i=0}^{\infty}$ if the inequalities $d(f^i(x), x_i) \leq \epsilon$, fix $\epsilon, \delta > 0$ and $i \in \mathbb{Z}$, holds.

Let $\mathcal{X}^{\mathrm{Z}}$ be the compact metric space of all two –sided sequences.

Definition 2.1. [9]

Let $(\mathcal{X}, d)$ be a compact metric space and $f: (\mathcal{X}, d) \rightarrow (\mathcal{X}, d)$ be a continuous map, f is said to be inverse shadowing property denoted by ISP (resp., positive inverse shadowing ISP^+) if for any $\epsilon > 0$ there exists $\delta > 0$ where for any $x \in \mathcal{X}$ and any δ-method $\emptyset : \mathcal{X} \rightarrow \mathcal{X}^z$, there is $s \in \mathcal{X}$ where $d(f^k(x), \emptyset_k(s)) < \epsilon$, For all $k \in \mathbb{Z}$

3- Main Theorems:

In this section, we state and show the main results about maps that have the inverse shadowing property in a metric space $(\mathcal{X}, d)$ and $(\mathcal{Y}, d)$.

Theorem 3.1

If f and $g: (\mathcal{X}, d) \rightarrow (\mathcal{X}, d)$ are maps on $(\mathcal{X}, d)$ have the inverse shadowing property, then $g \circ f$ has the inverse shadowing property.

Proof:

Suppose f have the inverse shadowing property by definition if for any $\epsilon > 0$ there exists $\delta_1 > 0$ where for any $x \in \mathcal{X}$ and any δ −method $\emptyset_1: \mathcal{X} \rightarrow \mathcal{X}^{\mathrm{Z}}$ there is $s_1 \in \mathcal{X}$ such that $d\left(f^k(x), \emptyset_{1k}(s_1)\right) < \epsilon$, for all $k \in \mathbb{Z}$.

Since g have the inverse shadowing property so there exists $\delta_2 > 0$ where for any $y = f(x) \in \mathcal{X}$ since f onto there exists $x \in \mathcal{X}$ and any δ − method $\emptyset_2 : \mathcal{X} \rightarrow \mathcal{X}^{\mathrm{Z}}$ there is $s_2 \in \mathcal{X}$ where $d\left(g^k(f(x)), \emptyset_{2k}(s_2)\right) < \epsilon_2$, for all $k \in \mathbb{Z}$.

We choose $\delta = \min\{\delta_1, \delta_2\}$ where for any $w = g \circ f(x) \in \mathcal{X}$ and any δ −method $\emptyset = \emptyset_2 \circ \emptyset_1$: $\mathcal{X} \rightarrow \mathcal{X}^{\mathrm{Z}}$, there is $s = s_2 \circ s_1 \in \mathcal{X}$ where

$d\left((g \circ f)^k(x), \emptyset_k(s)\right) < \epsilon$, for all $k \in Z$.

Hence $g \circ f$ has the inverse shadowing property. ■

Corollary 3.2

If $f : (\mathcal{X}, d) \rightarrow (\mathcal{X}, d)$ be a map. If f have the inverse shadowing property, then f^k has the inverse shadowing property for every $k \in \mathbb{N}$.

Proof:

We can prove this result by Induction Law of Theorem 3.1 . ■

Theorem 3.3

Let $(\mathcal{X}, d_1)$ and $(\mathcal{Y}, d_2)$ be two metric spaces , $f: (\mathcal{X}, d_1) \to (\mathcal{X}, d_1)$ and $g: (\mathcal{Y}, d_2) \to (\mathcal{Y}, d_2)$ be maps. Then $f \approx_h g$ if f have the inverse shadowing property , if and only if g have the inverse shadowing property .

Proof: $\Longrightarrow$

Suppose f have the inverse shadowing property by definition if for any $\epsilon_1 > 0$ there exists $\delta_1 > 0$ where for any $x \in \mathcal{X}$ and any δ −method $\emptyset_1: \mathcal{X} \to \mathcal{X}^{\mathbb{Z}}$ there is $s_1 \in \mathcal{X}$ where

$d\left(f^k(x)\ ,\ \emptyset_{1k}(s_1)\right) < \epsilon_1$, for all $k \in \mathbb{Z}$.

Since h is homeomorphism for any $y = h(x) \in \mathcal{Y}$ and any δ −method $h(\emptyset_1): \mathcal{Y} \to \mathcal{Y}^{\mathbb{Z}}$ there is $h(s) \in \mathcal{Y}$ where

$d_2((h \circ f)^k(x),\ \emptyset_1 \circ h\,(s)) < \epsilon$, for all $k \in \mathbb{Z}$.

Hence g has the inverse shadowing property .

$\Longleftarrow$

Since $h^{-1}: \mathcal{Y} \to \mathcal{X}$ also a homeomorphism this imply $\approx_{h^{-1}} f$, If g has the inverse shadowing property . Then by the first part of proof .

Hence f has the inverse shadowing property .

Hence $f \approx_h g$ have the inverse shadowing property . ■

Theorem 3.4

Let $(\mathcal{X}, d_1)$ and $(\mathcal{Y}, d_2)$ be two metric space , $f: (\mathcal{X}, d_1) \to (\mathcal{X}, d_1)$ and $g: (\mathcal{Y}, d_2) \to (\mathcal{Y}, d_2)$ be maps. If f and g have the inverse shadowing property if and only if $f \times g$ have the inverse shadowing property .

Proof:

Suppose f have the inverse shadowing property by definition if a given $\epsilon > 0$ there exists $\delta_1 > 0$ where for any $x \in \mathcal{X}$ and any δ −method $\emptyset_1: \mathcal{X} \to \mathcal{X}^{\mathbb{Z}}$ there is $s_1 \in \mathcal{X}$ where

$d\left(f^k(x)\ ,\ \emptyset_{1k}(s_1)\right) < \epsilon_1$, for all $k \in \mathbb{Z}$.

Since g have the inverse shadowing property so there exists $\delta_2 > 0$ where for any $y \in \mathcal{Y}$ and any δ −method $\emptyset_2: \mathcal{Y} \to \mathcal{Y}^{\mathbb{Z}}$ there is $s_2 \in \mathcal{Y}$ where

$d\left(g^k(y)\ ,\ \emptyset_{2k}(s_2)\right) < \epsilon_2$, for all $k \in \mathbb{Z}$.

We choose and $\delta = \max\{\delta_1, \delta_2\}$ where for any $w = x \times y \in \mathcal{X} \times \mathcal{Y}$ an any δ −method $\emptyset = \emptyset_1 \times \emptyset_2: (\mathcal{X} \times \mathcal{Y}) \to (\mathcal{X} \times \mathcal{Y})^{\mathbb{Z}}$, there is $s = s_1 \times s_2 \in \mathcal{X} \times \mathcal{Y}$ where

$d\left((f \times g)^k(w)\ ,\ \emptyset_k(s)\right) < \epsilon$, for all $k \in \mathbb{Z}$.

Hence $f \times g$ has the inverse shadowing property.

We Consider $(\mathcal{X} \times \mathcal{Y})^{\mathbb{Z}} = \mathcal{X}^{\mathbb{Z}} \times \mathcal{Y}^{\mathbb{Z}}$

Now, we state and prove the main results about maps have the inverse shadowing property in a metric space $(\mathbb{R}^n, d)$.

Theorem 3.5

Let $f, g : (\mathbb{R}^n, d) \to (\mathbb{R}^n, d)$ be maps .if f *and* g have the inverse shadowing property , Then $f + g$ have the inverse shadowing property.

Proof:

Suppose that f have the inverse shadowing property so for a given $\epsilon > 0$ there exists $\delta_1 > 0$ where for any $x \in \mathbb{R}^n$ and any δ −method $\emptyset_1: \mathbb{R}^n \to (\mathbb{R}^n)^{Z}$ there is $s_1 \in \mathbb{R}^n$ such that

$d\left(f^k(x) , \emptyset_{1k}(s_1)\right) < \epsilon_1$, for all $k \in \mathbb{Z}$.

Since g have the inverse shadowing property so there exists $\delta_2 > 0$ where for any $y \in \mathbb{R}^n$ and any δ −method $\emptyset_2: \mathbb{R}^n \to (\mathbb{R}^n)^{Z}$ there is $s_2 \in \mathbb{R}^n$ where

$d\left(g^k(y) , \emptyset_{2k}(s_2)\right) < \epsilon_2$, for all $k \in \mathbb{Z}$.

We choose and $\delta = \max\{\delta_1, \delta_2\}$ where for any $w = x + y \in \mathbb{R}^n$ and any δ −method $\emptyset = \emptyset_1 + \emptyset_2: \mathbb{R}^n \to (\mathbb{R}^n)^{Z}$, there is $s = s_1 + s_2 \in \mathbb{R}^n$ where

$d\left((f + g)^k(w) , \emptyset_k(s)\right) < \epsilon$, for all $k \in \mathbb{Z}$.

Hence $f + g$ have the inverse shadowing property .

Theorem 3.6

Let $f, g : (\mathbb{R}^n, d) \to (\mathbb{R}^n, d)$ be maps .if f *and* g have the inverse shadowing property , Then $f \cdot g$ have the inverse shadowing property .

Proof:

Suppose that f have the inverse shadowing property so for a given $\epsilon > 0$ there exists $\delta_1 > 0$ where for any $x \in \mathbb{R}^n$ and any δ −method $\emptyset_1: \mathbb{R}^n \to (\mathbb{R}^n)^{Z}$ there is $s_1 \in \mathbb{R}^n$ where

$d\left(f^k(x) , \emptyset_{1k}(s_1)\right) < \epsilon_1$, for all $k \in \mathbb{Z}$.

Since g have the inverse shadowing property so there exists $\delta_2 > 0$ where for any $y \in \mathbb{R}^n$ and any δ −method $\emptyset_2: \mathbb{R}^n \to (\mathbb{R}^n)^{Z}$ there is $s_2 \in \mathbb{R}^n$ where

$d\left(g^k(y) , \emptyset_{2k}(s_2)\right) < \epsilon_2$, for all $k \in \mathbb{Z}$.

Choose $\delta = \max\{\delta_1, \delta_2\}$ where for any $w = x \cdot y \in \mathbb{R}^n$ and any δ −method $\emptyset = \emptyset_1 \cdot \emptyset_2: \mathbb{R}^n \to (\mathbb{R}^n)^{Z}$, there is $s = s_1 \cdot s_2 \in \mathbb{R}^n$ where

$d\left((f \cdot g)^k(w) , \emptyset_k(s)\right) < \epsilon$, for all $k \in \mathbb{Z}$.

Hence $f \cdot g$ have the inverse shadowing property .

CONFLICT OF INTERESTS

There are no conflicts of interest

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بعض الخصائص العامة لخاصية معكوس الظل

الخلاصة

سوف ندرس خاصية معكوس الظل, تم برهان انه لو كان لدينا الدالتين $f,\ g$ على الفضاء المتري $(\mathcal{X}\ ,d\ )$ كلاهما تمتلك خاصية معكوس الظل فان $g\circ f,\ f^{k}\ ,f\approx_{h}\ g\ ,\ f\times g$ تمتلك خاصية معكوس الظل. اذا كانت هاتان الدالتين معرفتين على الفضاء $(\mathbb{R}^{n},d)$ و تمتلكان خاصية معكوس الظل فان $f.\ g\ ,\ f+g$ تمتلكان خاصية معكوس الظل.

الكلمات الدالة: المسار المحذوف, خاصية الظل, معكوس خاصية الظل.